Set Class List

Reginald Bain

This table was originally created for Stefan Kostka and Dorothy Payne, *Tonal Harmony: With An Introduction to Twentieth-Century Music*, 5th ed. (New York: McGraw Hill, 2004), see Appendix C *Set Class List*.

The 8-column table below lists all set classes (cardinality 3 through 9, inclusive) formed under T_n and/or T_nI (T_n/T_nI).[1] [2] The first and sixth columns list **Forte names** (FN) in increasing order. The second and seventh columns list **prime forms**. Prime forms are calculated using the Rahn algorithm.[3] The third and eighth columns list **interval-class vectors** (IC VECTOR). The interval-class vector of a set remains invariant under T_n/T_nI. Some Forte names include a Z (e.g., 4-Z15). The Z indicates that another set class listed in the table has the same ic vector. Set classes that share the same interval vector (e.g., 4-Z15 & 4-Z29) are said to **Z-related**. Inclusion of the Z in the Forte name is optional. The fourth column of the table gives the number of **distinct forms** (DF) of the set class. Most set classes have 24 distinct forms: 12 under T_n, and 12 under T_nI. When nothing appears in the DF column, the set class has 24 distinct forms. Set classes with fewer than 24 distinct forms are said to be **symmetrical**. These sets map onto themselves under T_n (at a level other than T_0) and/or T_nI. The **degree of symmetry** (DOS) entry in the fifth column is given using Straus notation (*x*, *y*): where *x* is the degree of **transpositional symmetry**, and *y* is the degree of **inversional symmetry**.[4] Nothing appears in in the DOS column in the trivial case where the DOS is 1,0. The relationship between the number of DF and DOS may be expressed using the following formula: $DF = 24/(x + y)$. Note that spaces have been added between the members of all prime forms and ic vectors to make them easier to read, search, and compare. The intervening spaces should not be used when notating prime forms and ic vectors. After Straus (2016) the symbols T and E have been substituted for 10 and 11, respectively, in the prime forms. Another common substitution scheme is A = 10 and B = 11 (Morris 1987).

Each row of the Set Class List is based on the **complement relation**. By way of example, Figure 1 shows the first row of the table. The pc set (C, C♯, D) is a member of set class 3-1 (012). The **literal complement** of (C, C♯, D) is (D♯, E, F, F♯, G, G♯, A, A♯, B), i.e., the pc set consisting of all other members of the **aggregate**.[5] The literal complement of pc set *X* will always have 12–*n* members, where *n* is the cardinality of *X*. The 9-member literal complement of (C, C♯, D) belongs to set class 9-1 (012345678). Complementary set classes have the same number of distinct forms and same degree of symmetry, so 3-1 and 9-1 may be listed on the same row of the table. Members of 9-1 are said to be the **abstract complements** of the members of 3-1, and vice versa. Complementary set classes also have proportional ic vectors.[6] Because hexachords partition the aggregate into two equal-sized parts, hexachordal set classes have some unique properties. For example, some of them are self-complementary. In the listing of hexachords, notice that nothing appears in sixth, seventh and eighth columns of the **self-complementary hexachords**.[7] After Morris (2001), we call the proportional ic vector property mentioned above the **complement theorem**. Its corollary the **hexachord theorem** states: if a hexachord is not self-complementary it must be Z-related to its complement.

Figure 1. First row of the Set Class List with columns labeled

					Listing for the complementary set class		
FN	**PRIME FORM**	**IC VECTOR**	**DF**	**DOS**	**FN**	**PRIME FORM**	**IC VECTOR**
3-1	(0 1 2)	2 1 0 0 0 0	12	1,1	9-1	(0 1 2 3 4 5 6 7 8)	8 7 6 6 6 3
Forte name	*Prime form*	*Interval-class vector*	*Distinct forms*	*Degree of symmetry*	*Forte name*	*Prime form*	*Interval-class vector*

[1] This table is designed for use in BAIN MUSC 525 *Post-Tonal Theory*. It is based on the following sources: Forte 1973, Appendix 1: Prime Forms and Vectors of Pitch-Class Sets, pp. 179-181; Rahn 1980, Table II, pp. 140-143; and Straus 2016, List of Set Classes, pp. 378-381. For a historical point of view on such tables, see Schuijer 2008.

[2] Straus (2016) uses the operation I_n instead of T_nI.

[3] Rahn 1980. *PC Polygon Assistant* (Bain 2016) uses the Rahn algorithm to calculate the normal form and prime form.

[4] Straus 2016, p. 103 and pp. 110-111.

[5] Ibid., 116. The term *aggregate* refers to the universal set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}, where C=0, C♯/D♭=1, D=2, …, B=11.

[6] Ibid., 118. For example, 9-1's ic vector 876663 is related to 3-1's ic vector 210000 in the following manner: subtract the cardinality difference between the two set classes (i.e., 9 – 3 = 6) from each ic entry of the former to obtain the latter – except for the ic6 entry, where (due to the symmetrical nature of the tritone) we must subtract the cardinality difference divided by 2 (i.e., 6/2 = 3).

[7] Ibid., 119.

TRICHORDS

FN	PRIME FORM	IC VECTOR	DF	DOS
3-1	(0 1 2)	2 1 0 0 0 0	12	1,1
3-2	(0 1 3)	1 1 1 0 0 0		
3-3	(0 1 4)	1 0 1 1 0 0		
3-4	(0 1 5)	1 0 0 1 1 0		
3-5	(0 1 6)	1 0 0 0 1 1		
3-6	(0 2 4)	0 2 0 1 0 0	12	1,1
3-7	(0 2 5)	0 1 1 0 1 0		
3-8	(0 2 6)	0 1 0 1 0 1		
3-9	(0 2 7)	0 1 0 0 2 0	12	1,1
3-10	(0 3 6)	0 0 2 0 0 1	12	1,1
3-11	(0 3 7)	0 0 1 1 1 0		
3-12	(0 4 8)	0 0 0 3 0 0	4	3,3

TETRACHORDS

FN	PRIME FORM	IC VECTOR	DF	DOS
4-1	(0 1 2 3)	3 2 1 0 0 0	12	1,1
4-2	(0 1 2 4)	2 2 1 1 0 0		
4-3	(0 1 3 4)	2 1 2 1 0 0	12	1,1
4-4	(0 1 2 5)	2 1 1 1 1 0		
4-5	(0 1 2 6)	2 1 0 1 1 1		
4-6	(0 1 2 7)	2 1 0 0 2 1	12	1,1
4-7	(0 1 4 5)	2 0 1 2 1 0	12	1,1
4-8	(0 1 5 6)	2 0 0 1 2 1	12	1,1
4-9	(0 1 6 7)	2 0 0 0 2 2	6	2,2
4-10	(0 2 3 5)	1 2 2 0 1 0	12	1,1
4-11	(0 1 3 5)	1 2 1 1 1 0		
4-12	(0 2 3 6)	1 1 2 1 0 1		
4-13	(0 1 3 6)	1 1 2 0 1 1		
4-14	(0 2 3 7)	1 1 1 1 2 0		
4-Z15	(0 1 4 6)	1 1 1 1 1 1		
4-16	(0 1 5 7)	1 1 0 1 2 1		
4-17	(0 3 4 7)	1 0 2 2 1 0	12	1,1
4-18	(0 1 4 7)	1 0 2 1 1 1		
4-19	(0 1 4 8)	1 0 1 3 1 0		
4-20	(0 1 5 8)	1 0 1 2 2 0	12	1,1
4-21	(0 2 4 6)	0 3 0 2 0 1	12	1,1
4-22	(0 2 4 7)	0 2 1 1 2 0		
4-23	(0 2 5 7)	0 2 1 0 3 0	12	1,1
4-24	(0 2 4 8)	0 2 0 3 0 1	12	1,1
4-25	(0 2 6 8)	0 2 0 2 0 2	6	2,2
4-26	(0 3 5 8)	0 1 2 1 2 0	12	1,1
4-27	(0 2 5 8)	0 1 2 1 1 1		
4-28	(0 3 6 9)	0 0 4 0 0 2	3	4,4
4-Z29	(0 1 3 7)	1 1 1 1 1 1		

NONACHORDS

FN	PRIME FORM	IC VECTOR
9-1	(0 1 2 3 4 5 6 7 8)	8 7 6 6 6 3
9-2	(0 1 2 3 4 5 6 7 9)	7 7 7 6 6 3
9-3	(0 1 2 3 4 5 6 8 9)	7 6 7 7 6 3
9-4	(0 1 2 3 4 5 7 8 9)	7 6 6 7 7 3
9-5	(0 1 2 3 4 6 7 8 9)	7 6 6 6 7 4
9-6	(0 1 2 3 4 5 6 8 T)	6 8 6 7 6 3
9-7	(0 1 2 3 4 5 7 8 T)	6 7 7 6 7 3
9-8	(0 1 2 3 4 6 7 8 T)	6 7 6 7 6 4
9-9	(0 1 2 3 5 6 7 8 T)	6 7 6 6 8 3
9-10	(0 1 2 3 4 6 7 9 T)	6 6 8 6 6 4
9-11	(0 1 2 3 5 6 7 9 T)	6 6 7 7 7 3
9-12	(0 1 2 4 5 6 8 9 T)	6 6 6 9 6 3

OCTACHORDS

FN	PRIME FORM	IC VECTOR
8-1	(0 1 2 3 4 5 6 7)	7 6 5 4 4 2
8-2	(0 1 2 3 4 5 6 8)	6 6 5 5 4 2
8-3	(0 1 2 3 4 5 6 9)	6 5 6 5 4 2
8-4	(0 1 2 3 4 5 7 8)	6 5 5 5 5 2
8-5	(0 1 2 3 4 6 7 8)	6 5 4 5 5 3
8-6	(0 1 2 3 5 6 7 8)	6 5 4 4 6 3
8-7	(0 1 2 3 4 5 8 9)	6 4 5 6 5 2
8-8	(0 1 2 3 4 7 8 9)	6 4 4 5 6 3
8-9	(0 1 2 3 6 7 8 9)	6 4 4 4 6 4
8-10	(0 2 3 4 5 6 7 9)	5 6 6 4 5 2
8-11	(0 1 2 3 4 5 7 9)	5 6 5 5 5 2
8-12	(0 1 3 4 5 6 7 9)	5 5 6 5 4 3
8-13	(0 1 2 3 4 6 7 9)	5 5 6 4 5 3
8-14	(0 1 2 4 5 6 7 9)	5 5 5 5 6 2
8-Z15	(0 1 2 3 4 6 8 9)	5 5 5 5 5 3
8-16	(0 1 2 3 5 7 8 9)	5 5 4 5 6 3
8-17	(0 1 3 4 5 6 8 9)	5 4 6 6 5 2
8-18	(0 1 2 3 5 6 8 9)	5 4 6 5 5 3
8-19	(0 1 2 4 5 6 8 9)	5 4 5 7 5 2
8-20	(0 1 2 4 5 7 8 9)	5 4 5 6 6 2
8-21	(0 1 2 3 4 6 8 T)	4 7 4 6 4 3
8-22	(0 1 2 3 5 6 8 T)	4 6 5 5 6 2
8-23	(0 1 2 3 5 7 8 T)	4 6 5 4 7 2
8-24	(0 1 2 4 5 6 8 T)	4 6 4 7 4 3
8-25	(0 1 2 4 6 7 8 T)	4 6 4 6 4 4
8-26	(0 1 3 4 5 7 8 T)	4 5 6 5 6 2
8-27	(0 1 2 4 5 7 8 T)	4 5 6 5 5 3
8-28	(0 1 3 4 6 7 9 T)	4 4 8 4 4 4
8-Z29	(0 1 2 3 5 6 7 9)	5 5 5 5 5 3

PENTACHORDS

FN	PRIME FORM	IC VECTOR	DF	DOS
5-1	(0 1 2 3 4)	4 3 2 1 0 0	12	1,1
5-2	(0 1 2 3 5)	3 3 2 1 1 0		
5-3	(0 1 2 4 5)	3 2 2 2 1 0		
5-4	(0 1 2 3 6)	3 2 2 1 1 1		
5-5	(0 1 2 3 7)	3 2 1 1 2 1		
5-6	(0 1 2 5 6)	3 1 1 2 2 1		
5-7	(0 1 2 6 7)	3 1 0 1 3 2		
5-8	(0 2 3 4 6)	2 3 2 2 0 1	12	1,1
5-9	(0 1 2 4 6)	2 3 1 2 1 1		
5-10	(0 1 3 4 6)	2 2 3 1 1 1		
5-11	(0 2 3 4 7)	2 2 2 2 2 0		
5-Z12	(0 1 3 5 6)	2 2 2 1 2 1	12	1,1
5-13	(0 1 2 4 8)	2 2 1 3 1 1		
5-14	(0 1 2 5 7)	2 2 1 1 3 1		
5-15	(0 1 2 6 8)	2 2 0 2 2 2	12	1,1
5-16	(0 1 3 4 7)	2 1 3 2 1 1		
5-Z17	(0 1 3 4 8)	2 1 2 3 2 0	12	1,1
5-Z18	(0 1 4 5 7)	2 1 2 2 2 1		
5-19	(0 1 3 6 7)	2 1 2 1 2 2		
5-20	(0 1 5 6 8)	2 1 1 2 3 1		
5-21	(0 1 4 5 8)	2 0 2 4 2 0		
5-22	(0 1 4 7 8)	2 0 2 3 2 1	12	1,1
5-23	(0 2 3 5 7)	1 3 2 1 3 0		
5-24	(0 1 3 5 7)	1 3 1 2 2 1		
5-25	(0 2 3 5 8)	1 2 3 1 2 1		
5-26	(0 2 4 5 8)	1 2 2 3 1 1		
5-27	(0 1 3 5 8)	1 2 2 2 3 0		
5-28	(0 2 3 6 8)	1 2 2 2 1 2		
5-29	(0 1 3 6 8)	1 2 2 1 3 1		
5-30	(0 1 4 6 8)	1 2 1 3 2 1		
5-31	(0 1 3 6 9)	1 1 4 1 1 2		
5-32	(0 1 4 6 9)	1 1 3 2 2 1		
5-33	(0 2 4 6 8)	0 4 0 4 0 2	12	1,1
5-34	(0 2 4 6 9)	0 3 2 2 2 1	12	1,1
5-35	(0 2 4 7 9)	0 3 2 1 4 0	12	1,1
5-Z36	(0 1 2 4 7)	2 2 2 1 2 1		
5-Z37	(0 3 4 5 8)	2 1 2 3 2 0	12	1,1
5-Z38	(0 1 2 5 8)	2 1 2 2 2 1		

SEPTACHORDS

FN	PRIME FORM	IC VECTOR
7-1	(0 1 2 3 4 5 6)	6 5 4 3 2 1
7-2	(0 1 2 3 4 5 7)	5 5 4 3 3 1
7-3	(0 1 2 3 4 5 8)	5 4 4 4 3 1
7-4	(0 1 2 3 4 6 7)	5 4 4 3 3 2
7-5	(0 1 2 3 5 6 7)	5 4 3 3 4 2
7-6	(0 1 2 3 4 7 8)	5 3 3 4 4 2
7-7	(0 1 2 3 6 7 8)	5 3 2 3 5 3
7-8	(0 2 3 4 5 6 8)	4 5 4 4 2 2
7 -9	(0 1 2 3 4 6 8)	4 5 3 4 3 2
7-10	(0 1 2 3 4 6 9)	4 4 5 3 3 2
7-11	(0 1 3 4 5 6 8)	4 4 4 4 4 1
7-Z12	(0 1 2 3 4 7 9)	4 4 4 3 4 2
7-13	(0 1 2 4 5 6 8)	4 4 3 5 3 2
7-14	(0 1 2 3 5 7 8)	4 4 3 3 5 2
7-15	(0 1 2 4 6 7 8)	4 4 2 4 4 3
7-16	(0 1 2 3 5 6 9)	4 3 5 4 3 2
7-Z17	(0 1 2 4 5 6 9)	4 3 4 5 4 1
7-Z18	(0 1 4 5 6 7 9)	4 3 4 4 4 2
7-19	(0 1 2 3 6 7 9)	4 3 4 3 4 3
7-20	(0 1 2 5 6 7 9)	4 3 3 4 5 2
7-21	(0 1 2 4 5 8 9)	4 2 4 6 4 1
7-22	(0 1 2 5 6 8 9)	4 2 4 5 4 2
7-23	(0 2 3 4 5 7 9)	3 5 4 3 5 1
7-24	(0 1 2 3 5 7 9)	3 5 3 4 4 2
7-25	(0 2 3 4 6 7 9)	3 4 5 3 4 2
7-26	(0 1 3 4 5 7 9)	3 4 4 5 3 2
7-27	(0 1 2 4 5 7 9)	3 4 4 4 5 1
7-28	(0 1 3 5 6 7 9)	3 4 4 4 3 3
7-29	(0 1 2 4 6 7 9)	3 4 4 3 5 2
7-30	(0 1 2 4 6 8 9)	3 4 3 5 4 2
7-31	(0 1 3 4 6 7 9)	3 3 6 3 3 3
7-32	(0 1 3 4 6 8 9)	3 3 5 4 4 2
7-33	(0 1 2 4 6 8 T)	2 6 2 6 2 3
7-34	(0 1 3 4 6 8 T)	2 5 4 4 4 2
7-35	(0 1 3 5 6 8 T)	2 5 4 3 6 1
7-Z36	(0 1 2 3 5 6 8)	4 4 4 3 4 2
7-Z37	(0 1 3 4 5 7 8)	4 3 4 5 4 1
7-Z38	(0 1 2 4 5 7 8)	4 3 4 4 4 2

HEXACHORDS

FN	PRIME FORM	IC VECTOR	DF	DOS	FN	PRIME FORM	IC VECTOR
6-1	(0 1 2 3 4 5)	5 4 3 2 1 0	12	1,1			
6-2	(0 1 2 3 4 6)	4 4 3 2 1 1					
6-Z3	(0 1 2 3 5 6)	4 3 3 2 2 1			6-Z36	(0 1 2 3 4 7)	4 3 3 2 2 1
6-Z4	(0 1 2 4 5 6)	4 3 2 3 2 1	12	1,1	6-Z37	(0 1 2 3 4 8)	4 3 2 3 2 1
6-5	(0 1 2 3 6 7)	4 2 2 2 3 2					
6-Z6	(0 1 2 5 6 7)	4 2 1 2 4 2	12	1,1	6-Z38	(0 1 2 3 7 8)	4 2 1 2 4 2
6-7	(0 1 2 6 7 8)	4 2 0 2 4 3	6	2,2			
6-8	(0 2 3 4 5 7)	3 4 3 2 3 0	12	1,1			
6-9	(0 1 2 3 5 7)	3 4 2 2 3 1					
6-Z10	(0 1 3 4 5 7)	3 3 3 3 2 1			6-Z39	(0 2 3 4 5 8)	3 3 3 3 2 1
6-Z11	(0 1 2 4 5 7)	3 3 3 2 3 1			6-Z40	(0 1 2 3 5 8)	3 3 3 2 3 1
6-Z12	(0 1 2 4 6 7)	3 3 2 2 3 2			6-Z41	(0 1 2 3 6 8)	3 3 2 2 3 2
6-Z13	(0 1 3 4 6 7)	3 2 4 2 2 2	12	1,1	6-Z42	(0 1 2 3 6 9)	3 2 4 2 2 2
6-14	(0 1 3 4 5 8)	3 2 3 4 3 0					
6-15	(0 1 2 4 5 8)	3 2 3 4 2 1					
6-16	(0 1 4 5 6 8)	3 2 2 4 3 1					
6-Z17	(0 1 2 4 7 8)	3 2 2 3 3 2			6-Z43	(0 1 2 5 6 8)	3 2 2 3 3 2
6-18	(0 1 2 5 7 8)	3 2 2 2 4 2					
6-Z19	(0 1 3 4 7 8)	3 1 3 4 3 1			6-Z44	(0 1 2 5 6 9)	3 1 3 4 3 1
6-20	(0 1 4 5 8 9)	3 0 3 6 3 0	4	3,3			
6-21	(0 2 3 4 6 8)	2 4 2 4 1 2					
6-22	(0 1 2 4 6 8)	2 4 1 4 2 2					
6-Z23	(0 2 3 5 6 8)	2 3 4 2 2 2	12	1,1	6-Z45	(0 2 3 4 6 9)	2 3 4 2 2 2
6-Z24	(0 1 3 4 6 8)	2 3 3 3 3 1			6-Z46	(0 1 2 4 6 9)	2 3 3 3 3 1
6-Z25	(0 1 3 5 6 8)	2 3 3 2 4 1			6-Z47	(0 1 2 4 7 9)	2 3 3 2 4 1
6-Z26	(0 1 3 5 7 8)	2 3 2 3 4 1	12	1,1	6-Z48	(0 1 2 5 7 9)	2 3 2 3 4 1
6-27	(0 1 3 4 6 9)	2 2 5 2 2 2					
6-Z28	(0 1 3 5 6 9)	2 2 4 3 2 2	12	1,1	6-Z49	(0 1 3 4 7 9)	2 2 4 3 2 2
6-Z29	(0 2 3 6 7 9)	2 2 4 2 3 2	12	1,1	6-Z50	(0 1 4 6 7 9)	2 2 4 2 3 2
6-30	(0 1 3 6 7 9)	2 2 4 2 2 3	12	2,0			
6-31	(0 1 4 5 7 9)	2 2 3 4 3 1					
6-32	(0 2 4 5 7 9)	1 4 3 2 5 0	12	1,1			
6-33	(0 2 3 5 7 9)	1 4 3 2 4 1					
6-34	(0 1 3 5 7 9)	1 4 2 4 2 2					
6-35	(0 2 4 6 8 T)	0 6 0 6 0 3	2	6,6			

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