

Straus Chapter 3

Some Additional Properties and Relationships

Joseph N. Straus, *Introduction to Post-Tonal Theory*, 4th ed. (New York: Norton, 2016), pp. 95-132.

TERMS & CONCEPTS

COMMON-TONE THEOREMS

Common tones under T_n (§ 3.1, p. 96)

The number of common tones under T_n may be determined from the ic vector. For example, the pc set [C, C#, F#, G], whose ic vector is 200022, produces the following number (#) of common tones at (@) each level of T_n :

@	T_{11}	T_{10}	T_9	T_8	T_7	T_6
#	2	0	0	0	2	2 * 2
@	T_1	T_2	T_3	T_4	T_5	T_6

Because the tritone maps onto itself under T_6 , you have to multiply the ic vector's ic₆ entry by 2.

Common tones under I_n (§ 3.3, p. 103)

Index matrix, or addition table

Index scoreboard

Index vector

SYMMETRY AND SET CLASS (SC)

Transpositional symmetry (§ 3.2, p. 100)

Self-mapping operation

Degree of transpositional symmetry

Transpositionally symmetrical SCs (p. 101)

Inversional symmetry (§ 3.4, p. 107)

Degree of inversional symmetry

Inversionally symmetrical SCs:

- at one level of I_n
 - at more than one level of I_n (p. 111)
- Intervallic palindrome, or mirror image
 See also: The 12 inversional axes (p. 240)
- Mirror or *axis* name for each I_n is: $\frac{n}{2}$ to $\frac{n}{2} + 6$

Pitch space symmetry (§ 3.4.2, p. 108)

Center of symmetry as pitch centricity

Some special set classes

Augmented triad: 3-12 (048); Diminished seventh chord: 4-28 (0369); All interval tetrachords: 4-Z15 (0146) & 4-Z29 (0137); All trichord hexachord: 6-Z17 (012478); Petrushka chord: 6-30 (013679); Diatonic hexachord: 6-32 (024579), etc. See also the Ch. 5 referential collections: 5-35, 6-20, 6-35, 7-35 & 8-28.

DOS & DF (§ 3.5, p. 112)

Degree of symmetry (DOS)

Straus (x, y) notation, where:

- x is the # of T_n self-mapping operations
- y is the # of I_n self-mapping operations

Distinct forms (DF): $DF = 24/(x + y)$

PROPERTIES & RELATIONSHIPS

Set type under:

- Cardinality (c)
- T_n , T_n/I_n , T_nM , T_nMI , etc.

Similarity relations

Z relation (§ 3.6, p. 112)

The all-interval tetrachords:

4-Z15 (0146) & 4-Z29 (0137)

Z-correspondents

Complement relation (§ 3.7, p. 115)

Aggregate

Any collection containing all 12 pitch classes

Literal complement

Abstract complement

Complement Theorem

- Proportional distribution of ic (§ 3.7.1)
- Same degree of symmetry (§ 3.7.3)

Hexachord Theorem

- Every hexachordal set class is *self-complementary*, or is *Z-related to its complement*

Interval Vector properties

Unique multiplicity of ic (Ex. 1-22, p. 16)

Maximum ic (e.g., Ex. 3-26 Max. ic 4, p. 118)

Minimum ic

Inclusion relation (§ 3.8, p. 121)

Subsets & supersets

Power set, has 2^c members

Literal subset/superset relations

Abstract subset/superset relations

Inclusion lattice (§ 3.8.2, p. 122)

- Subset classes

Transpositional combination (§ 3.9, p. 124)

TC property

Contour Relations (§ 3.10, p. 126)

CSEG and CSEG-class

EXAMPLE. Stockhausen, *Klavierstück IX*

PC set:



$$X = (1, 6, 7, 0)$$

Normal form: [0, 1, 6, 7]

Set class¹

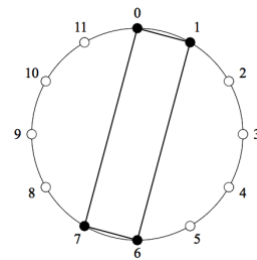
Set class: 4-9 (0167)

IC vector: 200022

DOS: 2, 2

CINT1: 1-5-1-5

Polygon notation



Complement:

Literal complement: $X' = [2, 3, 4, 5, 8, 9, 10, 11]$

Abstract complement: 8-9 (01236789) 644464

Common tones under T_n and the ic vector:²

@	T ₀	T ₁	T ₂	T ₃	T ₄	T ₅	T ₆	T ₇	T ₈	T ₉	T ₁₀	T ₁₁
# of common tones	4	2	0	0	0	2	4	2	0	0	0	2

Common tones under I_n and the index matrix (or addition table) and scoreboard:³

	0	1	6	7
0	0	1	6	7
1	1	2	7	8
6	6	7	0	1
7	7	8	1	2

@	I ₀	I ₁	I ₂	I ₃	I ₄	I ₅	I ₆	I ₇	I ₈	I ₉	I ₁₀	I ₁₁
# of common tones	2	4	2	0	0	0	2	4	2	0	0	0

Index vector:⁴ 242000242000

[0, 1, 6, 7] maps onto itself at T₀, T₆, I₁ & I₇, thus its degree of symmetry (DOS) is: (2, 2)

Literal and Abstract Subsets⁵

Literal subsets: {0, 1, 6, 7}, {0, 1, 6}, {0, 1, 7}, {0, 6, 7}, {1, 6, 7}, {0, 1}, {0, 6}, {0, 7}, {1, 6}, {1, 7}, {6, 7}, {0}, {1}, {6}, {7}, {}

Subset classes: All 4-note subsets belong to set class (0167); All 3-note subsets belong to set class (016); The 2-note subsets belong to set classes (01), (05) & (06).

References

Straus, Joseph N. 2016. *Introduction to Post-Tonal Theory*, 4th ed. New York: Norton.

Morris, Robert. 1987. *Composition with Pitch Classes*. New Haven: Yale University Press.

Software

Bain, Reginald. 2016. *PC Polygon Assistant*. Available online at: <<https://reginaldbain.com/software.html>>

¹ DOS is the *degree of symmetry* (see p. 119). CINT1 is the *cyclic interval array* of the prime form (see Morris 1987, CINT₁).

² If an entry equals the cardinality of the set, the set *maps onto itself* at (@) that level of T_n. Every set maps onto itself at T₀.

³ The *index matrix* shows the inversionsal *sums* in a *c*-by-*c* matrix. The scoreboard shows the total number (#) of occurrences of each sum in the matrix, which corresponds to the number of common tones @ each level of I_n.

⁴ We define the *index vector* to be a listing (without commas) of the number of common tones produced under I₀, I₁, I₂, ..., I₁₁.

⁵ The number of literal subsets is 2^c, where *c* is the *cardinality* of the pc set.